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An effective computational strategy has been developed for the analysis of large and complex structures. The strategy is based on generating the response of the complex structures using large perturbations from that of a lower-order (simpler) model associated with a simpler structure (or a simpler mathematical/discrete model of the original structure). The three key elements of the strategy are: (a) mixed (or primitive variable) formulation with the fundamental unknowns consisting of generalized displacements and stress parameters; (b) operator splitting, or a reduction method to relate the arrays and degrees of freedom of the original complex structure to those of the simpler system; and (c) efficient iterative process for the generation of the response of the complex structure starting from that of the simpler system. The strategy has been successfully applied to a number of linear and nonlinear stress analysis problems, free vibration and nonlinear structural dynamics problems. The strategy was also used to obtain accurate transverse stresses in laminated composite plates and shells, using the two-dimensional first-order shell (plate) theory as the simpler model.

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Final Report

Grant No. AFOSR 87-0115

IMPROVED COMPUTATIONAL STRATEGY FOR PREDICTING THE RESPONSE OF COMPLEX SYSTEMS

March 1990

ABSTRACT

An effective computational strategy has been developed for the analysis of large and complex structures. The strategy is based on generating the response of the complex structure using *large perturbations* from that of a lower-order (simpler) model associated with a simpler structure (or a simpler mathematical/discrete model of the original structure). The three key elements of the strategy are: (a) mixed (or primitive variable) formulation with the fundamental unknowns consisting of generalized displacements and stress parameters; (b) operator splitting, or a reduction method to relate the arrays and degrees of freedom of the original complex structure to those of the simpler system; and (c) efficient iterative process for the generation of the response of the complex structure starting from that of the simpler system. The strategy has been successfully applied to a number of linear and nonlinear stress analysis problems, free vibration and nonlinear structural dynamics problems. The strategy was also used to obtain accurate transverse stresses in laminated composite plates and shells, using the two-dimensional first-order shell (plate) theory as the simpler model.

INTRODUCTION

The work under this grant focused on the development of an effective computational strategy for the solution of large-scale structural problems. The strategy developed combines the following three major characteristics:

1) gives physical insight about the response;

2) helps in assessing the adequacy of the computational model; and

3) is highly efficient.

The first characteristic is a direct consequence of the basic idea of the strategy; namely, generating the response of a complex structure using large perturbations from that of a simpler structure (or a sequence of simpler structures). Examples of complex and corresponding simpler structural systems are given in Table 1.

The second characteristic is significant because of the importance of assessing the reliability of response predictions (i.e., the agreement between the response predictions of the computational model and those of the actual structure). In the absence of reliable and practical

error measures, information about the sensitivity of the response to modeling details can help in identifying the regions of questionable accuracy, and in the adaptive refinement of the computational model. Sensitivity information can be obtained by using more than one mathematical and/or discrete model of the structure. Only the lower-order model equations are solved. The higher-order model arrays are used to generate the sensitivity derivatives.

High computational efficiency is achieved by reducing the time required for the accurate numerical simulation of the response which, in turn, requires:

a) reducing the number of degrees of freedom used in the initial discretization (the initial discrete model is, in many cases, dictated by the topology rather than the complexity of the response);

b) exploiting the major features of the new and emerging computing systems (viz., vector, parallel, and AI capabilities). AI-knowledge-based systems are used in the initial selection and adaptive refinement of the model as well as in the selection of numerical algorithms. Work in the last period focused only on reducing the number of degrees of freedom used in the initial discretization.

The three key elements of the strategy are as follows: 1) use of mixed (or primitive variable) formulation with the fundamental unknowns consisting of generalized displacements and stress parameters; 2) a novel operator splitting technique which allows the restructuring the governing equations of the complex structure in such a manner as to delineate the contributions to the simpler system; 3) application of an iterative procedure or a reduction method for the efficient generation of the response of the complex structure starting from that of the simpler system.

Two general approaches have been developed for selecting the simpler model and establishing the relations between the original and simpler models, namely, *hierarchical modeling*; and *decomposition or partitioning strategy*. Successful applications of the strategy to static, free vibration and dynamic response problems of composite panels have been reported in the publications listed subsequently. Also, application has been made of the strategy to three-dimensional analysis of multilayered composite panels. Both stress and free vibration problems have been considered.

BASIC IDEA AND KEY ELEMENTS OF STRATEGY

Application of the strategy developed to the static stress and vibration analyses of both symmetric and unsymmetric structures has been reported in Refs. 1 to 7. Herein, the application to nonlinear static problems is outlined. For convenience, simpler structure is chosen as a structure with symmetric geometry and symmetric (orthotropic) material characteristics. The unsymmetric nonlinear response of the original anisotropic structure is approximated by a linear combination of symmetric and antisymmetric response vectors (which are uncoupled in the simpler structure). Each of the symmetric and antisymmetric vectors is obtained by using only a fraction of the degrees of freedom of the original finite element model, as briefly described subsequently.

1. Governing Finite Element Equations

A total Lagrangian formulation is used, and the structure is discretized by using two-field mixed finite element models. The loading is controlled by a single parameter q. The governing finite element equations describing the large deflection, nonlinear response of the structure can be written in the following compact form:

$$\begin{Bmatrix} f_H(H,X,q) \\ f_X(H,X,q) \end{Bmatrix} = \begin{bmatrix} -[F] & [S] \\ [S]^t & \cdot \end{bmatrix} \begin{Bmatrix} H \\ X \end{Bmatrix} + \begin{Bmatrix} G(X) \\ M(H,X) \end{Bmatrix} - q \begin{Bmatrix} \cdot \\ P \end{Bmatrix} = 0 \qquad (1)$$

where $\{H\}$ is the vector of stress-resultant parameters; $\{X\}$ is the vector of nodal displacements; $[F]$ is the global flexibility matrix; $[S]$ is the strain-displacement matrix; $\{P\}$ is the normalized load vector; $\{G(X)\}$ and $\{M(H,X)\}$ are vectors of the nonlinear terms for the structure; superscript t denotes transposition; and a dot (•) refers to a zero submatrix, or a zero subvector.

The determination of the deformed configuration of the structure corresponding to specified values of the parameter q is accomplished by solving the nonlinear system of algebraic equations, equations (1), using an incremental-iterative technique such as the Newton-Raphson method. The recursion formula for the rth iteration, can be written in the following form:

$$\begin{bmatrix} -[F] & [S]+[\bar{S}^{(r)}] \\ [S]^t+[\bar{S}^{(r)}]^t & [K^{(r)}] \end{bmatrix} \begin{Bmatrix} \Delta H \\ \Delta X \end{Bmatrix}^{(r)} = -\begin{Bmatrix} f_H \\ f_X \end{Bmatrix}^{(r)} \qquad (2)$$

$$\begin{Bmatrix} H \\ X \end{Bmatrix}^{(r+1)} = \begin{Bmatrix} H \\ X \end{Bmatrix}^{(r)} + \begin{Bmatrix} \Delta H \\ \Delta X \end{Bmatrix}^{(r)} \tag{3}$$

where $$[\bar{S}] = \left[\frac{\partial G_i}{\partial X_j} \right] \tag{4}$$

and the range of i is 1 to the total number of stress-resultant parameters in the model.

In order to reduce the cost of generating the deformed configurations of the structure, reduction methods have been developed for substantially reducing the number of degrees of freedom used in the initial discretization. These methods are based on successive applications of the finite element method and the classical Bubnov-Galerkin technique. The finite element method is used to generate a few global approximation vectors (or modes). The Bubnov-Galerkin technique is then used to approximate the nonlinear equations, equations (1), by a small set of nonlinear equations in the amplitudes of these modes. An effective set of modes was found to be the *path derivatives* (i.e., the various-order derivatives of the response quantities with respect to a control parameter (such as load, displacement, or arc-length parameter in the solution space). The equations used in evaluating the path derivatives are obtained by successive differentiation of the original nonlinear equations, equations (1), with respect to the control parameter. The left-hand side matrix in these equations is the same as that of equations (2). The details of application of reduction methods to the generation of the equilibrium path are given in Refs. 8, 9 and 10, and involve the following basic steps:

i) Evaluation of the path derivatives, and formation of the reduced equations which approximate the original nonlinear equations

ii) Generation of the approximate solutions associated with different values of the control parameter, using an incremental/iterative approach, in conjunction with the reduced equations.

iii) Sensing the error resulting from the use of the approximate reduced equations. Whenever the error exceeds the prescribed tolerance, an iterative procedure is used to generate an improved solution, a new (updated) set of path derivatives, and a new set of reduced equations.

The most time-consuming steps of the procedure are those associated with visiting the original, full system of equations, namely, generation of an initial (or improved) nonlinear

solution, and evaluation of path derivatives. Hence, the reduction in the size of the analysis model used in these steps, as outlined in the succeeding subsection, can reduce the total computational effort.

2. Operator Splitting

To simplify the nonlinear analysis of anisotropic structures, each of the response vectors in equations (1) and (2) is decomposed into symmetric and antisymmetric components (with the same type of symmetry as that exhibited by the structure; or as desired in order to reduce the size of the analysis model).

Each of the arrays in equations (1) and (2) is partitioned into the sum of the array associated with the corresponding symmetric response plus correction terms. The resulting equations are embedded into a single parameter family of equations of the form given subsequently:

$$\begin{Bmatrix} f_H \\ f_X \end{Bmatrix} = \left(\begin{bmatrix} -[F]_o & [S]_o \\ [S]_o^t & \cdot \end{bmatrix} + \lambda \begin{bmatrix} -[F]_\lambda & [S]_\lambda \\ [S]_\lambda^t & \cdot \end{bmatrix} \right) \begin{Bmatrix} H \\ X \end{Bmatrix} + \begin{Bmatrix} G_o \\ M_o \end{Bmatrix}$$

$$+ \lambda \begin{Bmatrix} G_\lambda \\ M_\lambda \end{Bmatrix} - q \left(\begin{Bmatrix} \cdot \\ P_o \end{Bmatrix} + \lambda \begin{Bmatrix} \cdot \\ P_\lambda \end{Bmatrix} \right) = 0 \tag{5}$$

The recursion formula for the rth iteration, equations (4), can be cast in the following form:

$$\left(\begin{bmatrix} -[F]_o & [S]_o + [\bar{S}^{(r)}]_o \\ [S]_o^t + [\bar{S}^{(r)}]_o^t & [K^{(r)}]_o \end{bmatrix} + \lambda \begin{bmatrix} -[F]_\lambda & [S]_\lambda + [\bar{S}^{(r)}]_\lambda \\ [S]_\lambda^t + [\bar{S}^{(r)}]_\lambda^t & [K^{(r)}]_\lambda \end{bmatrix} \right) \begin{Bmatrix} \Delta H \\ \Delta X \end{Bmatrix}^{(r)}$$

$$= - \begin{Bmatrix} f_H \\ f_X \end{Bmatrix}_o^{(r)} - \lambda \begin{Bmatrix} f_H \\ f_X \end{Bmatrix}_\lambda^{(r)} \tag{6}$$

In equations (5) and (6), λ is a tracing parameter which identifies all the correction terms (terms with subscript λ) to the arrays associated with the symmetric response (terms with subscript o). For $\lambda=0$, the structure has a symmetric response, and for $\lambda=1$, the response is unsymmetric.

The equations used in evaluating the path derivatives can also be embedded in a single-

parameter family of equations and cast in a form similar to that of equations (6).

3. Generation of the Response Vectors

The solutions of equations (5) and (6) at $\lambda=0$ are used in conjunction with either a) a reduction method, or b) preconditioned conjugate gradient (PCG) technique to generate the corresponding solutions at $\lambda=1$. This results in reducing the size of the analysis model required for the generation of these solutions. A detailed description of the PCG technique is given in Refs. 2 and 6. In this section a brief outline of the reduction method is given.

The method is based on generating a few global approximation vectors at $\lambda=0$. The solutions of equations (5) and (6) are then expressed as linear combinations of these global approximation vectors in the following transformation:

$$\begin{Bmatrix} H \\ X \end{Bmatrix} = \begin{bmatrix} \Gamma_H \\ \Gamma_X \end{bmatrix} \{\psi\} \tag{7}$$

where $[\Gamma_H]$ and $[\Gamma_X]$ are transformation matrices and $\{\psi\}$ is a vector of undetermined coefficients (amplitudes of global approximation vectors) which are functions of λ.

An effective choice for the global approximation vectors (the columns of the matrices $[\Gamma]$) consists of the solution corresponding to $\lambda=0$ and its various-order derivatives with respect to λ (evaluated at $\lambda=0$), i.e.,

$$[\Gamma_H] = \left[\{H\} \; \left\{\frac{\partial H}{\partial \lambda}\right\} \; \left\{\frac{\partial^2 H}{\partial \lambda^2}\right\} \cdots \left\{\frac{\partial^{r-1} H}{\partial \lambda^{r-1}}\right\} \right]_{\lambda=0} \tag{8}$$

$$[\Gamma_X] = \left[\{X\} \; \left\{\frac{\partial X}{\partial \lambda}\right\} \; \left\{\frac{\partial^2 X}{\partial \lambda^2}\right\} \cdots \left\{\frac{\partial^{r-1} X}{\partial \lambda^{r-1}}\right\} \right]_{\lambda=0} \tag{9}$$

The total number of approximation vectors is r, which is considerably smaller than the total number of degrees of freedom in the model.

The equations used in generating the global approximation vectors are obtained by successive differentiation of the original equations (Equations (5) and (6)) with respect to λ. For symmetric loading, the vectors $\{H\}_{\lambda=0}$ and $\{X\}_{\lambda=0}$ and all their even-order derivatives with respect to λ are symmetric, and all the odd-order derivatives are antisymmetric. The Bubnov-Galerkin technique is then used to approximate the original equations (equations (5) or (6)) by a

reduced system of equations in the unknown parameters (components of the vector $\{\psi\}$). The details of application of the strategy to eigenvalue problems is described in Refs. 3 and 7. Its application to nonlinear, large deflection problems is described in Refs. 1 and 6.

IMPORTANCE OF THE COMPLETED STUDY

Because of the requirements of high performance, light weight and economy for future flight vehicles, and the associated stringent design criteria, the prediction of the response of these structures is likely to require more sophisticated analysis models than has heretofore been done. Also, analysis is needed to reduce the dependence on extensive and expensive testing which is frequently component or mission oriented. Therefore, there is a need for innovative solution methodologies and effective computational strategies, such as the one developed under the present grant, for handling large-scale structural and coupled problems. The work completed under the present grant is a first step towards filling this need and coupling the physics of the problem with the solution strategy.

REFERENCES

1. Noor, A. K. and Peters, J. M., "Nonlinear Analysis of Anisotropic Panels," AIAA Journal, Vol. 24, No. 9, 1986, pp. 1545-1553.
2. Noor, A. K. and Peters, J. M., "Preconditioned Conjugate Gradient Technique for the Analysis of Symmetric Anisotropic Structures," International Journal for Numerical Methods in Engineering, Vol. 24, 1987, pp. 2057-2070.
3. Noor, A. K. and Whitworth, S. L., "Model-Size Reduction for the Buckling and Vibration Analyses of Anisotropic Panels," Journal of Engineering Mechanics Division, ASCE, Vol. 113, No. 2, 1987, pp. 170-185.
4. Noor, A. K. and Peters, J. M., "Model Size Reduction Technique for the Analysis of Symmetric Anisotropic Structures," Engineering Computations, Vol. 2, No. 4, 1985, pp. 285-292.
5. Noor, A. K. and Whitworth, S. L., "Computational Strategy for Analysis of Quasi-symmetric Structures," Journal of Engineering Mechanics Division, ASCE, Vol. 114, No. 3, 1988, pp. 456-477.

6. Noor, A. K. and Peters, J. M., "Model-Size Reduction for the Nonlinear Dynamic Analysis of Quasi-symmetric Structures," Engineering Computations, Vol. 4, 1987, pp. 178-189.

7. Noor, A. K. and Whitworth, S. L., "Vibration Analysis of Quasi-symmetric Structures," Finite Elements in Analysis and Design, Vol. 3, 1987, pp. 257-276.

8. Noor, A. K., "On Making Large Nonlinear Problems Small," Computer Methods in Applied Mechanics and Engineering, Vol. 34, 1982, pp. 955-985.

9. Noor, A. K. and Peters, J. M., "Bifurcation and Postbuckling Analysis of Laminated Composite Plates Via Reduced Basis Technique," Computer Methods in Applied Mechanics and Engineering, Vol. 29, 1981, pp. 271-295.

10. Noor, A. K. and Peters, J. M., "Recent Advances in Reduction Methods for Instability Analysis of Structures," Computers and Structures, Vol. 16, Nos. 1-4, 1983, pp. 67-80.

Table 1 - Examples of Complex and Corresponding Simpler Strucural Systems

Original Structure	Simpler Structure
• Structure with complicated geometry	• Structure with simpler geometry (see Fig. 1)
• Anisotropic structure	• Orthotropic structure
• Symmetric structure with unsymmetric boundary conditions	• Symmetrized structure with SYMMETRIZED boundary conditions
• Three-dimensional elasticity model of laminated shell	• Two-dimensional first-order shear deformation model
• Stiffened shell	• Unstiffened shell
• Fine (enriched) grid model	• Coarse grid model

PUBLICATIONS AND PRESENTATIONS

A total of 18 publications have been written, and 5 presentations have been made under this Grant. A list of the publications and presentations are given subsequently. Also, abstracts of the papers, along with the full length papers are enclosed.

<u>Publications</u>

1. Noor, A. K. and Whitworth, S. L., "Computational Strategy for Analysis of Quasi-symmetric Structures," Journal of Engineering Mechanics Division, ASCE, Vol. 114, No. 3, 1988, pp. 456-477.
2. Noor, A. K. and Peters, J. M., "Model-Size Reduction for the Nonlinear Dynamic Analysis of Quasi-symmetric Structures," Engineering Computations, Vol. 4, 1987, pp. 178-189.
3. Noor, A. K. and Peters, J. M., "Nonlinear Dynamic Analysis of Quasi-symmetric Anisotropic Structures," Computers and Structures, Vol. 27, 1987, pp. 1-12.
4. Noor, A. K., "Parallel Processing in Finite Element Structural Analysis," in Parallel Computations and Their Impact on Mechanics, ASME AMD Vol. 86, 1987, pp. 253-277.
5. Noor, A. K. and Peters, J. M., "A Computational Strategy for Making Complicated Structural Problems Simple," Computer Methods in Applied Mechanics and Engineering, Vol. 71, No. 2, Nov. 1988, pp. 167-182.
6. Noor, A. K. and Burton, W. S., "Assessment of Shear Deformation Theories for Multilayered Composite Plates," Applied Mechanics Reviews, Vol. 42, No. 1, 1989, pp. 1-13.
7. Noor, A. K. and Burton, W. S., "Stress and Free Vibration Analyses of Multilayered Composite Plates," Journal of Composite Structures, Vol. 11, No. 2, 1989, pp. 183-204.
8. Noor, A. K. and Burton, W. S., "Analysis of Multilayered Anisotropic Plates - A New Look at an Old Problem," Proceedings of the 30th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, April 3-5, 1989, Mobile, AL.
9. Noor, A. K. and Burton, W. S., "Assessment of Computational Models for Multilayered Anisotropic Plates," Composite Structures, Vol. 14 (to appear).
10. Noor, A. K. and Peters, J. M., "Stress, Vibration, and Buckling of Multilayered Cylinders," Journal of Structural Engineering, Vol. 115, No. 1, Jan. 1989, pp. 69-88.
11. Noor, A. K. and Burton, W. S., "Three-Dimensional Solutions for Antisymmetrically

Laminated Anisotropic Plates," Journal of Applied Mechanics (to appear).

12. Noor, A. K. and Peters, J. M., "Potential of Mixed Formulations for Advanced Analysis Systems," Computers and Structures (to appear).
13. Noor, A. K. and Peters, J. M., "Buckling and Postbuckling Analyses of Laminated Anisotropic Structures," International Journal for Numerical Methods in Engineering (to appear).
14. Noor, A. K. and Peters, J. M., "Novel Computational Strategies for Solution of Large-Scale Structural Problems," Proceedings of the IUTAM/IACM Symposium on Discretization Metods in Structural Mechanics, Vienna, June 5-9, 1989, Springer, New York, 1989.
15. Noor, A. K., Burton, W. S. and Peters, J. M., "Assessment of Computational Models for Multilayered Composite Cylinders," International Journal of Solids and Structures (to appear).
16. Noor, A. K. and Burton, W. S., "Assessment of Computational Models for Multilayered Composite Shells," Applied Mechanics Reviews (to appear).
17. Noor, A. K. and Peters, J. M., "Buckling and Postbuckling of Thin-Walled Composite Stiffeners," International Journal of Space Structures (to appear).
18. Noor, A. K., Burton, W. S. and Peters, J. M., "Predictor-Corrector Procedures for Stress and Free Vibration Analyses of Multilayered Composite Plates and Shells," Computer Methods in Applied Mechanics and Engineering (to appear).

<u>Presentations</u>

1. Noor, A. K., "Large-Scale Structural Analysis on Parallel Computers," invited talk at Virginia Polytechnic Institute and State University, March 23-25, 1987.
2. Noor, A. K., "Computational Strategy for Large-Scale Structural Problems," invited talk at University of Illinois at Urbana-Champaign, April 20-22, 1987.
3. Noor, A. K., "Parallel Processing in Finite Element Structural Analysis," Symposium on Parallel Computations and Their Impact on Mechanics, ASME Winter Annual Meeting, Boston, MA, Dec. 14-15, 1987.
4. Noor, A. K., "Solution Strategies for Large-Scale Structural Problems," International Conference on Computational Engineering Science, Atlanta, GA, April 10-14, 1988.

5. Noor, A. K. and Peters, J. M., "A Computational Strategy for Making Complicated Structural Problems Simple," International Conference on Computational Engineering Science, Atlanta, GA, April 10-14, 1988.

COMPUTATIONAL STRATEGY FOR ANALYSIS OF QUASI-SYMMETRIC STRUCTURES

By Ahmed K. Noor,[1] Member, ASCE, and Sandra L. Whitworth[2]

ABSTRACT: An efficient computational strategy is presented for reducing the cost of generating the response of quasi-symmetric structures. The three key elements of the strategy are: a) Use of mixed finite element models having independent shape functions for the internal forces (stress resultants) and generalized displacements with the internal forces allowed to be discontinuous at interelement boundaries; b) operator splitting, or additive decomposition of the different arrays in the governing equations into the contributions to a symmetrized response plus correction terms; and c) application of a preconditioned conjugate gradient technique to generate the unsymmetric response as the sum of symmetric and antisymmetric modes, each obtained using approximately half the degrees of freedom of the original model. The preconditioning matrix is taken to be the matrix associated with the symmetrized response. The effectiveness of the proposed strategy is demonstrated by means of two numerical examples of an anisotropic shallow panel with a quadrilateral planform, and an anisotropic conical panel. Also, the potential of the proposed strategy for solving nonlinear problems of quasi-symmetric structures is discussed.

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Model-size reduction for the non-linear dynamic analysis of quasi-symmetric structures

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(*Received March 1987*)

ABSTRACT

A computational procedure is presented for the efficient non-linear dynamic analysis of quasi-symmetric structures. The procedure is based on *approximating the unsymmetric response vectors, at each time step, by a linear combination of symmetric and antisymmetric vectors, each obtained using approximately half the degrees of freedom of the original model.* A mixed formulation is used with the fundamental unknowns consisting of the internal forces (stress resultants), generalized displacements and velocity components. The spatial discretization is done by using the finite element method, and the governing semi-discrete finite element equations are cast in the form of first-order non-linear ordinary differential equations. The temporal integration is performed by using implicit multistep integration operators. The resulting non-linear algebraic equations, at each time step, are solved by using iterative techniques. The three key elements of the proposed procedure are: (a) use of mixed finite element models with independent shape functions for the stress resultants, generalized displacements, and velocity components and with the stress resultants allowed to be discontinuous at interelement boundaries; (b) operator splitting, or restructuring of the governing discrete equations of the structure to delineate the contributions to the symmetric and antisymmetric vectors constituting the response; and (c) use of a two-level iterative process (with nested iteration loops) to generate the symmetric and antisymmetric components of the response vectors at each time step. The top- and bottom-level iterations (outer and inner iterative loops) are performed by using the Newton Raphson and the preconditioned conjugate gradient (PCG) techniques, respectively. The effectiveness of the proposed strategy is demonstrated by means of a numerical example and the potential of the strategy for solving more complex non-linear problems is discussed.

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NONLINEAR DYNAMIC ANALYSIS OF QUASI-SYMMETRIC ANISOTROPIC STRUCTURES

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Abstract—An efficient computational strategy is presented for the nonlinear dynamic analysis of quasi-symmetric anisotropic structures. A mixed formulation is used with the fundamental unknowns consisting of stress resultants, generalized displacements and velocity components. The governing semi-discrete finite element equations consist of a mixed system of algebraic and ordinary differential equations. The temporal integration of the differential equations is performed by using an explicit half-station central difference method. The three key elements of the strategy are: (a) use of mixed finite element models with independent shape functions for the stress resultants, generalized displacements and velocity components and with the stress resultants allowed to be discontinuous at interelement boundaries; (b) operator splitting, or additive decomposition of the different arrays in the governing equations into the contributions to a symmetrized response plus correction terms; and (c) application of a preconditioned conjugate gradient technique to generate the unsymmetric response of the structure, at each time step, as the sum of symmetric and antisymmetric modes, each obtained using approximately half the degrees of freedom of the original model. The preconditioning matrix is taken to be the matrix associated with the symmetrized response.

The effectiveness of the proposed strategy is demonstrated by means of a numerical example and the potential of the proposed strategy for solving more complex nonlinear problems is discussed.

Engineering with Computers 3, 225–241 (1988)

Engineering with Computers

Parallel Processing in Finite Element Structural Analysis

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Abstract. A brief review is made of the fundamental concepts and basic issues of parallel processing. Discussion focuses on mechanisms for parallel processing, construction and implementation of parallel numerical algorithms, performance evaluation of parallel processing machines and numerical algorithms, and parallelism in finite element computations. A novel partitioning strategy is outlined for maximizing the degree of parallelism on computers with a small number of powerful processors.

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A COMPUTATIONAL STRATEGY FOR MAKING COMPLICATED STRUCTURAL PROBLEMS SIMPLE

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An effective computational strategy is presented for the analysis of large and complex structures. The strategy is based on generating the response of the complex structure using *large perturbations* from that of a lower-order (simpler) model associated with a simpler structure (or a simpler mathematical/discrete model of the original structure). The three key elements of the strategy are: (a) mixed (or primitive variable) formulation with the fundamental unknowns consisting of generalized displacements and stress parameters; (b) operator splitting, or a reduction method to relate the arrays and degrees of freedom of the original complex structure to those of the simpler system; and (c) efficient iterative process for the generation of the response of the complex structure starting from that of the simpler system. The effectiveness of the proposed strategy is demonstrated by means of two numerical examples.

Assessment of shear deformation theories for multilayered composite plates

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A review is made of the different approaches used for modeling multilayered composite plates. Discussion focuses on different approaches for developing two-dimensional shear deformation theories; classification of two-dimensional theories based on introducing plausible displacement, strain and/or stress assumptions in the thickness direction; and first-order shear deformation theories based on linear displacement assumptions in the thickness coordinate. Extensive numerical results are presented showing the effects of variation in the lamination and geometric parameters of simply supported composite plates on the accuracy of the static and vibrational responses predicted by six different modeling approaches (based on two-dimensional shear deformation theories). The standard of comparison is taken to be the exact three-dimensional elasticity solutions. Some of the future directions for research on the modeling of multilayered composite plates are outlined.

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Stress and Free Vibration Analyses of Multilayered Composite Plates

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ABSTRACT

A two-phase computational procedure is presented for the accurate prediction of the vibration frequencies, stresses and deformations in multilayered composite plates. In the first phase a two-dimensional first-order shear deformation theory is used to predict the global response characteristics (vibration frequencies, 'average' through-the-thickness displacements and rotations) as well as the in-plane stress and strain components in the different layers. In the second phase, equilibrium equations and constitutive relations of the three-dimensional theory of elasticity are used to: (1) calculate the transverse stresses and strains as well as the transverse strain energy densities in the different layers; (2) provide better estimates for the composite shear correction factors; and (3) calculate corrected values for the vibration frequencies, displacements, and in-plane strains and stresses.

For simply supported plates the predictions of the proposed procedure are shown to be in close agreement with exact three-dimensional elasticity solutions for a wide range of lamination and geometric parameters. Also, the potential of the proposed procedure for use in conjunction with large-scale finite element models of composite structures is discussed.

ANALYSIS OF MULTILAYERED ANISOTROPIC PLATES - A NEW LOOK AT AN OLD PROBLEM

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Abstract

A study is made of the effects of variation in the lamination and geometric parameters of multilayered anisotropic (nonorthotropic) plates on the accuracy of the static and vibrational responses predicted by six modeling approaches, based on two-dimensional shear deformation theories. Two key elements distinguish the present study from previous studies reported in the literature: a) the standard of comparison is taken to be the *exact three-dimensional elasticity solutions*, and b) quantities compared are not limited to gross response characteristics (e.g., vibration frequencies, strain energy components, average through-the-thickness displacements and rotations), but include detailed, through-the-thickness, distributions of displacements, stresses and strain energy densities.

The modeling approaches considered include first-order shear-deformation theory (with five displacement parameters to characterize the deformation in the thickness direction); first-order theory with the transverse normal stresses and strains included (six displacement parameters); two higher-order theories (with 11 and 18 displacement parameters); a simplified higher-order theory (with 5 displacement parameters); and a predictor-corrector approach, used in conjunction with the first-order shear deformation theory (with five displacement parameters in the predictor phase)

Based on the numerical studies conducted, the predictor-corrector approach appears to be the most effective among the six modeling approaches considered. For antisymmetrically laminated rectangular plates the response quantities obtained by the predictor-corrector approach are shown to be in close agreement with exact three-dimensional elasticity solutions for a wide range of lamination and geometric parameters. The potential of this approach for predicting the response of multilayered anisotropic plates with complicated geometry is also discussed.

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Assessment of Computational Models for Multilayered Anisotropic Plates

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ABSTRACT

A study is made of the effects of variation in the lamination and geometric parameters of multilayered anisotropic (nonorthotropic) plates on the accuracy of the static and vibrational responses predicted by eight modeling approaches, based on two-dimensional shear-deformation theories. Two key elements distinguish the present study from previous studies reported in the literature: (1) the standard of comparison is taken to be the exact three-dimensional elasticity solutions, *and (2) quantities compared are not limited to gross response characteristics (e.g. vibration frequencies, strain energy components, average through-the-thickness displacements and rotations), but include detailed through-the-thickness distributions of displacements, stresses and strain energy densities.*

The modeling approaches considered include first-order shear-deformation theory (with five displacement parameters to characterize the deformation in the thickness direction); first-order theory with the transverse normal stresses and strains included (six displacement parameters); two higher-order theories (with 11 and 18 displacement parameters); a simplified higher-order theory (with five displacement parameters); discrete-layer theory (with piecewise linear variation of the in-plane displacements in the thickness direction); simplified discrete-layer theory with the continuity of transverse stresses imposed at layer interfaces to reduce the number of displacement parameters to five; and a predictor–corrector approach, used in conjunction with the first-order shear-deformation theory (with five displacement parameters in the predictor phase).

Based on the numerical studies conducted, the predictor–corrector approach appears to be the most effective among the eight modeling

approaches considered. For antisymmetrically laminated rectangular plates the response quantities obtained by the predictor-corrector approach are shown to be in close agreement with exact three-dimensional elasticity solutions for a wide range of lamination and geometric parameters. The potential of this approach for predicting the response of multilayered anisotropic plates with complicated geometry is also discussed.

STRESS, VIBRATION, AND BUCKLING OF MULTILAYERED CYLINDERS

By Ahmed K. Noor,[1] Member, ASCE, and Jeanne M. Peters[2]

ABSTRACT: An efficient computational procedure is presented for reducing the cost of the stress, free vibration, and buckling analyses of multilayered composite cylinders. The analytical formulation is based on the linear three-dimensional theory of elasticity. The cylinders are assumed to have simply supported curved edges, and the fibers of the different layers are either in the circumferential or longitudinal direction. The fundamental unknowns consist of the six stress components and the three displacement components of the cylinder. Each of the variables is expressed in terms of a double Fourier series in the longitudinal and circumferential coordinates, and a two-field mixed finite element model is used for the discretization in the thickness direction. The cylinder response associated with a range of Fourier harmonics in the longitudinal and circumferential directions is approximated by a linear combination of a few global approximation vectors, which are generated at particular values of the Fourier harmonics, within that range. The full equations of the finite element model are solved for only a single pair of Fourier harmonics, and the response corresponding to the other Fourier harmonics is generated using a reduced system of equations with considerably fewer degrees of freedom.

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Three-Dimensional Solutions for Antisymmetrically Laminated Anisotropic Plates

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Analytic three-dimensional elasticity solutions are presented for the stress and free vibration problems of multilayered anisotropic plates. The plates are assumed to have rectangular geometry and antisymmetric lamination with respect to the middle plane. A mixed formulation is used with the fundamental unknowns consisting of the six stress components and the three displacement components of the plate. Each of the plate variables is decomposed into symmetric and antisymmetric components in the thickness direction, and is expressed in terms of a double Fourier series in the Cartesian surface coordinates. Extensive numerical results are presented showing the effects of variation in the lamination and geometric parameters of composite plates on the importance of the transverse stress and strain components.

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POTENTIAL OF MIXED FORMULATIONS FOR ADVANCED ANALYSIS SYSTEMS

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Abstract—Two recent applications of the combined use of mixed formulation with splitting methodologies, and reduction methods are presented. Discussion focuses on how the combination can significantly improve the efficiency of the computational process and enhance the physical understanding of the response. Numerical examples are presented which demonstrate the unique features and potential of mixed formulations in advanced analysis systems.

BUCKLING AND POSTBUCKLING ANALYSES OF LAMINATED ANISOTROPIC STRUCTURES

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ABSTRACT

A review is given of recent advances in two aspects of the numerical simulation of the buckling and postbuckling responses of composite structures. The first aspect is exploiting nontraditional symmetries exhibited by composite structures; and strategies for reducing the size of the model and the cost of the buckling and postbuckling analyses in the presence of symmetry-breaking conditions (e.g., unsymmetry of the material, geometry, and/or loading). The second aspect pertains to the prediction of onset of local delamination in the postbuckling range and accurate determination of transverse shear stresses in the structure. The accuracy and effectiveness of the strategies developed are demonstrated by means of a numerical example.

Novel Computational Strategies for Solution of Large-Scale Structural Problems

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Summary
Novel computational strategies are presented for the analysis of large and complex structures. The strategies are based on generating the response of the complex structure using *large perturbations* from that of a simpler model, associated with a simpler structure (or a simpler mathematical/discrete model of the original structure). Numerical examples are presented to demonstrate the effectiveness of the strategies developed.

ASSESSMENT OF COMPUTATIONAL MODELS FOR MULTILAYERED COMPOSITE CYLINDERS

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ABSTRACT

A study is made of the effects of variation in the lamination and geometric parameters of multilayered composite cylinders on the accuracy of the static and vibrational responses predicted by eight modeling approaches, based on two-dimensional shear-deformation shell theories. The standard of comparison is taken to be the *exact* three-dimensional elasticity solutions, and the quantities compared include both the gross response characteristics (e.g., vibration frequencies, strain energy components, average through-the-thickness displacements and rotations); and detailed, through-the-thickness, distributions of displacements, stresses and strain energy densities.

Based on the numerical studies conducted, a predictor-corrector approach, used in conjunction with the first-order shear-deformation theory (with five displacement parameters in the predictor phase), appears to be the most effective among the eight modeling approaches considered. For multilayered orthotropic cylinders the response quantities obtained by the predictor-corrector approach are shown to be in close agreement with the exact three-dimensional elasticity solutions for a wide range of lamination and geometric parameters. The potential of this approach for predicting the response of multilayered shells with complicated geometry is also discussed.

BUCKLING AND POSTBUCKLING OF THIN-WALLED COMPOSITE STIFFENERS

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ABSTRACT

A review is given of the different types of symmetry exhibited by the prebuckling, buckling and postbuckling responses of composite stiffeners. A simple and efficient computational strategy is presented for generating both the postbuckling response and the sensitivity derivatives, with respect to lamination and material parameters, in the presence of symmetry-breaking conditions. The potential of the foregoing strategy for solving practical space structures problems is discussed, and its effectiveness is demonstrated by means of a numerical example of the postbuckling and sensitivity analyses of a composite stiffener with a zee-section subjected to uniform end shortening.

PREDICTOR-CORRECTOR PROCEDURES FOR STRESS AND FREE VIBRATION ANALYSES OF MULTILAYERED COMPOSITE PLATES AND SHELLS

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ABSTRACT

A study is made of two predictor-corrector procedures for the accurate determination of the global, as well as detailed, static and vibrational response characteristics of plates and shells. Both procedures use first-order shear deformation theory in the predictor phase, but differ in the elements of the computational model being adjusted in the corrector phase. The first procedure calculates *a posteriori* estimates of the composite correction factors and uses them to adjust the transverse shear stiffnesses of the plate (or shell). The second procedure calculates *a posteriori* the functional dependence of the displacement components on the thickness coordinate. The corrected quantities are then used in conjunction with the three-dimensional equations to obtain better estimates for the different response quantities. Extensive numerical results are presented showing the effects of variation in the geometric and lamination parameters for antisymmetrically laminated anisotropic plates, and simply supported multilayered orthotropic cylinders, on the accuracy of the linear static and free vibrational responses obtained by the predictor-corrector procedures. Comparison is also made with the solutions obtained by other computational models based on two-dimensional shear deformation theories. For each problem the standard of comparison is taken to be the analytic three-dimensional elasticity solution. The numerical examples clearly demonstrate the accuracy and effectiveness of the predictor-corrector procedures.